Online Appendix

Networks and Interethnic Cooperation

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1 Full Cooperation in Equilibrum

Full Cooperation in Equilibrium

This section contains the conditions for full cooperation under Network In-Group Policing, σ^{NWIGP} . σ^{NWIGP} is sequentially rational for all members of groups A and B if and only if, given r, p, T, g_A and g_B :

$$\delta^{T} \ge \frac{(\alpha - 1)(n - 1)}{\min_{i,j} \# N_{j-i}^{rT} (1 - p)(1 + \beta)},\tag{1}$$

and

$$\delta^{T} \ge \frac{\beta(n-1)}{\min_{i,j} \# N_{j-i}^{rT} (1-p)(1+\beta)}$$
(2)

for all $i, j \in A$ and $i, j \in B$, where $N_{j-i}^{rT} \equiv N_j^{rT}(g \setminus i) \cup j$.

Note that the conditions reduce to the conditions in Fearon and Laitin (1996) when the communication networks g^A and g^B are complete. In that case, $\#N_{j-i}^{rT} = n - 1$ for all $i, j \in A \cup B$ for any r and T. The proof, along with a discussion of beliefs that extend the behavior to sequential equilibrium, is as follows:

Proof. To establish sequential rationality, I will show that for any history and at any information set, all players prefer to comply with σ^{NWIGP} given the conditions above. Players' strategies respond to messages; specifically, they implement punishment in response to messages sent from victims of in-group deviations and observers of out-group deviations.

 $M_{i,t}$ is the set of messages *i* has received about the last *T* rounds by time *t*, and contains the identities of offenders who deserve punishment. If the network is incomplete, it could be that *i* has deviated from σ^{NWIGP} and yet some potential opponent *j* has not received a message saying so. To make the exposition clearer, call a player "in bad standing" if he has deviated from σ^{NWIGP} in the last *T* rounds and "in good standing" if he has complied with σ^{NWIGP} in the last *T* rounds.

A player *i* can be in bad standing and yet not face punishment from an opponent *j* in a particular round if *j* has not received a message indicating *i*'s bad standing; i.e. if $i \notin M_{j,t}$. A player's standing is determined by his use of messages, so a player always knows when he himself misbehaves and earns bad standing.¹ Additionally, a player who is in bad standing

¹For instance, *i* playing *d* in a pairing with some j about whom no message has been received, $j \notin M_{i,t}$, warrants punishment even if *j* so happened to be in bad standing. *j* can determine that *i* would not have known about his defection, and so spreads the word that *i* deserves punishment. In this scenario, *i* was

knows against whom he misbehaved and knows the network structure, so he knows who has received messages indicating his bad standing.

There are eight ways for an individual i to deviate from her strategy. When player i is in good standing:

- i Play d against a $j \notin M_{i,t}$
- ii Play c against a $j \in M_{i,t}$
- iii Play d in an out-group pairing

When player i is in bad standing:

- iv Play d with $j \notin M_{i,t}$ when $i \in M_{j,t}$
- v Play d with $j \notin M_{i,t}$ when $i \notin M_{j,t}$
- vi Play c with $j \in M_{i,t}$ when $i \in M_{j,t}$
- vii Play c with $j \in M_{i,t}$ when $i \notin M_{j,t}$

viii Play d in an out-group pairing

Assessing compliance with σ^{NWIGP} requires a lot of accounting detail for intermediate rounds that ends up falling out of the binding conditions. Recall from the article text that N_i^k is the k-neighborhood of player *i* in a network, which is the set of individuals reachable from *i* in paths of length *k* or shorter. The assumptions about message transmission and rate *r* result in a message sent from *i* reaching individuals N_i^{rl} in *l* rounds. The complement of this set (excluding *i*) will be denoted \overline{N}_i^{rl} . The number of individuals in these sets are $\#N_i^{rl}$ and $\#\overline{N}_i^{rl}$.

A player who deviates does not participate in the spread of the message about his own deviation. Any messages his victims send are effectively traveling through the network $g|_{A\setminus i}$, the subnetwork induced by everyone except i. When the need arises to specify the reach of i's victim j, I will use the notation N_{j-i}^{rl} , where the j-i indicates that this is j's reach in the network comprised of everyone except i. This is equivalent to, but slightly less cumbersome to write than, $N_j^{rl}(g\backslash i) \cup j$.

When a player assesses future costs and benefits of a decision made in t at a hypothesized history of play, an intermediate range of future periods are ambiguous to the player.

defecting and accidentally got lucky; the strategy does not reward this luck.

Specifically, *i* cannot know how many others about whom he will receive messages for the next few rounds.² Let \star denote parameters that are the result of a player's guess. $G_i^{l\star}$ is player *i*'s guess of the set of individuals about whom *i* will not have received a message by period *l* (will be thought to be in Good standing), and likewise $B_i^{l\star}$ is *i*'s guess of the set of individuals about whom *i* will be known to be in Bad standing). When a player knows this information rather than guesses it, it will be displayed without the \star . The binding conditions are independent of these guesses, precluding the need to precisely specify beliefs. Without loss of generality, the conditions will be presented for players in *A*.

1.1 Deviations via (i)

First consider deviations according to (i), in which a player in good standing plays d in a round with an in-group player j about whom i has received no message. The expected number of future punishers depends on j's network position. Expected payoffs for the next T-1 periods depend on guesses G^* and B^* ; after that, regardless of history, σ^{NWIGP} ensures that $G^* = A \setminus i$ (which has size n-1) and $B^* = \emptyset$. Complying with σ^{NWIGP} would yield:

$$1 + \sum_{l=1}^{T-1} \delta^l \left[\frac{1-p}{n-1} \left[\# G_i^{l\star} + \# B_i^{l\star} \alpha \right] + p \right] + \delta^T + \sum_{l=T+1}^{\infty} \delta^l$$

whereas deviating would yield:

$$\alpha + \sum_{l=1}^{T-1} \delta^{l} \left[\frac{1-p}{n-1} \left[\# (N_{j-i}^{rl} \cap G_{i}^{l\star})(-\beta) + \# (N_{j-i}^{rl} \cap B_{i}^{l\star})(0) + \# (\overline{N}_{j-i}^{rl} \cap G_{i}^{l\star}) + \# (\overline{N}_{j-i}^{rl} \cap B_{i}^{l\star}) \alpha \right] + p \right] + \delta^{T} \left[\frac{1-p}{n-1} \left[\# (N_{j-i}^{rl})(-\beta) + \# (\overline{N}_{j-i}^{rl}) \right] + p \right] + \sum_{l=T+1}^{\infty} \delta^{l}.$$

The expressions above simplify using the fact that $\#(N_{j-i}^{rl} \cap A \setminus i) = \#N_{j-i}^{rl}$. Complying in a round with j is thus weakly preferred to deviating when:

$$\alpha - 1 \leqslant \sum_{l=1}^{T-1} \frac{\delta^l (1-p)}{n-1} \left[\# (N_{j-i}^{rl} \cap G_i^{l\star})(1+\beta) + \# (N_{j-i}^{rl} \cap B_i^{l\star})\alpha] \right] + \frac{\delta^T (1-p)}{n-1} \left[\# N_{j-i}^{rT} (1+\beta)] \right].$$

²Though he does know this number is bounded by his rT-neighborhood, the maximum set of people i could ever receive messages about.

The sum in the condition contains terms for all but the final round of expected punishment and depends in each round on the number of individuals reached by j's message about whom i does not expect to have received a message by then, and the number of individuals reached by j's message about whom i does expect to have received a message by then. By the end of the punishment phase, everyone will be cooperative in compliance with σ^{NWIGP} , so the net cost of punishment in that round is only determined by the number of others who will be reached by j's message by then.

1.2 Deviations via (ii)

Deviations according to (ii), in which a player *i* in good standing plays *c* rather than *d* with a player about whom *i* has received a message, are trivially not preferred. Complying with σ^{NWIGP} yields α ; deviating earns 1, strictly less by assumption.

1.3 Deviations via (iii)

Deviating via (iii) entails player i in good standing playing d in an out-group pairing. The form of this defection is identical to (i), except that it is i's network position rather than an ingroup opponent's that matters (since out-group interactions are observable by assumption). The condition then becomes:

$$\alpha - 1 \leqslant \sum_{l=1}^{T-1} \frac{\delta^l (1-p)}{n-1} \left[\# (N_i^{rl} \cap G_i^{l\star})(1+\beta) + \# (N_i^{rl} \cap B_i^{l\star})\alpha \right] \right] + \frac{\delta^T (1-p)}{n-1} \left[\# N_i^{rT} (1+\beta) \right].$$

1.4 Deviations via (iv)

Consider an i in bad standing who is contemplating a defection via (iv), in which he would play d against a j about whom he has received no message. This deviation is similar in form to (i), with one important difference: i, being in bad standing, already expected some amount of punishment. His assessment of this deviation then depends on the net additional cost from punishment. This amount depends both on how recent his most recent past deviation was, as well as the identity of his past victim.

Call t^d the number of periods ago *i* most recently defected, and call *k* the victim of *i*'s defense in that round. This means in *t*, *i* faces $T - t^d$ more rounds of punishment from

that offense. A new offense today extends the punishment to t + T. Moreover, defecting on someone new, j, in t who is far away from his past victim k can increase the amount of punishment i expects even in the next $T - t^d$ rounds.

i's expected punishment from his defection against k that occurred t^d rounds ago depends on the network position of k.³ From the round under consideration until $T - t^d$ rounds into the future, *i* expects punishment for this offense which is a function of N_k . Specifically, his expected punishment *l* rounds from now depends on

$$N_{k-i}^{r(t^d+l)}.$$

His net expected punishment from defecting on j now also depends on j's network position. For the next $T - t^d$ rounds, his expected punishment depends on all those who receive a message from k and/or a message from j. That is, his expected punishment l rounds from now until $T - t^d$ depends on:

$$N_{k-i}^{r(t^d+l)} \cup N_{j-i}^{rl}$$

After that and until T rounds from now, his punishment depends on the message sent from j, N_{i-i}^{rl} . *i* then prefers to comply rather than deviate via (iv) against j so long as:

$$\beta \leqslant \sum_{l=1}^{T-t^d} \frac{\delta^l (1-p)}{n-1} \left[NW_G^{l\star}(-\beta) + \overline{NW}_G^{l\star} + \overline{NW}_B^{l\star} \alpha \right]$$
$$+ \sum_{l=T-t^d+1}^{T-1} \frac{\delta^l (1-p)}{n-1} \left[\# (N_{j-i}^{rl} \cap G_i^{l\star})(1+\beta) + \# (N_{j-i}^{rl} \cap B_i^{l\star}) \alpha \right]$$
$$+ \frac{\delta^T (1-p)}{n-1} \left[\# N_{j-i}^{rT}(1+\beta) \right]$$

where

³This implicitly assumes that k is a member of the in-group. The same condition results if we assume k is a member of the out-group and then use i's network position as the relevant consideration for the cost of his past defection.

$$\begin{split} NW_G^{l\star} &= \#(N_{j-i}^{rl} \cap N_{k-i}^{r(t^d+l)} \cap G_i^{l\star}) - \#(N_{j-i}^{rl} \cap G_i^{l\star}),\\ \overline{NW}_G^{l\star} &= \#\overline{(N_{j-i}^{rl} \cap N_{k-i}^{r(t^d+l)} \cap G_i^{l\star})} - \#\overline{(N_{j-i}^{rl} \cap G_i^{l\star})}\\ \overline{NW}_B^{l\star} &= \#\overline{(N_{j-i}^{rl} \cap N_{k-i}^{r(t^d+l)} \cap B_i^{l\star})} - \#\overline{(N_{j-i}^{rl} \cap B_i^{l\star})}. \end{split}$$

This condition can be further simplified.⁴ First, note that the condition is more difficult to satisfy as the right hand side decreases. The only term dependent on the identity of the first victim k is the first sum. This first sum takes its maximum when the neighborhoods around j and k are distinct; conversely, when new victim j is in the t^d -neighborhood of k, the condition is hardest to satisfy.

Consequently, one scenario in the set of binding cases is that in which *i* is contemplating defecting for a second time against the same person, i.e. when j = k. In that case, $N_{j-i}^{rl} \cap N_{k-i}^{r(t^d+l)} = N_{j-i}^{rl}$, and since intersection is associative, $NW_G^{l\star} = \overline{NW}_G^{l\star} = \overline{NW}_B^{l\star} = 0$. That is, in the binding case, an additional defection adds no cost to the first $T - t^d$ rounds of expected punishment. The binding condition becomes:

$$\beta \leqslant \sum_{l=T-t^d+1}^{T-1} \frac{\delta^l (1-p)}{n-1} \left[\# (N_{j-i}^{rl} \cap G_i^{l\star})(1+\beta) + \# (N_{j-i}^{rl} \cap B_i^{l\star})\alpha \right] + \frac{\delta^T (1-p)}{n-1} \left[\# N_{j-i}^{rT} (1+\beta) \right].$$

Furthermore, since the terms of the sum are all positive, the right hand side is minimized when the sum includes fewer periods. This means the binding case is one in which the second defection on k occurs immediately following the first, i.e. in which the old defection just occurred in the last period so that $t^d = 1$. In this binding case, the first T - 1 rounds of punishment would be the same, and the condition reduces further to

$$\beta \leq \frac{\delta^T (1-p)}{n-1} \left[\# N_{j-i}^{rT} (1+\beta) \right].$$

So long as the person considering defecting a second time against a player in back-to-back rounds can be disincentivized from doing so, no one wants to defect a second time against

⁴ The current form makes use of the following set operation: $\#(N_k \cap G) - \#((N_k \cup N_j) \cap G) = \#(N_k \cap N_j \cap G)) - \#(N_j \cap G)$.

anyone at any time.

1.5 Deviations via (v)

Deviations via (v), in which a player *i* plays *d* against a *j* for whom $i \notin M_{j,t}$, differ from (iv) only in the immediate gain. Expected net future punishment is the same. In this case, complying yields 1 immediately while deviating earns α , so the condition becomes:

$$\alpha - 1 \leqslant \frac{\delta^T (1 - p)}{n - 1} \left[\# N_{j-i}^{rT} (1 + \beta) \right].$$

1.6 Deviations via (vi) and (vii)

As with deviations via (ii), deviating by playing c against someone about whom a message has been received is trivially not preferred. In (vi) complying yields 0 while deviating yields $-\beta$; in (vii) complying yields α while deviating yields 1.

1.7 Deviations via (viii)

Similar to the comparison of (i) and (iii), the condition to disincentivize i from playing d in an out-group pairing is similar to the condition to disincentivize him from playing d against an in-group member about whom he has received no message and who has received no message about i (v). The difference is that, since the out-group interaction is public (and since no message sent by someone in the out-group reaches anyone in the in-group), the network position that matters is i's. The condition is the same as in (v) otherwise:

$$\alpha - 1 \leqslant \frac{\delta^T (1-p)}{n-1} \left[\# N_i^{rT} (1+\beta) \right].$$

1.8 Combining Conditions

Satisfying the condition to prevent deviations via (v) for all potential opponents j implies satisfying the condition to prevent deviations via (i). Additionally, satisfying the condition to prevent deviations via (viii) implies satisfying the condition to prevent deviations via (iii). Moreover, since $\{N_j^{rT}\}_{\forall j \in A} = \{N_i^{rT}\}_{\forall i \in A}$, since $g^A|_{A \setminus i} \subset g^A$, and since there are no isolates, satisfying condition (v) for all members of A implies satisfying condition (iii).⁵ Rearranging to place the discount factor on the left hand side, we have the two conditions stated above. Clearly the same conditions must be met for all players in B as well. If both conditions are satisfied for both groups, no player has an incentive to deviate from σ^{NWIGP} in any history, and the binding player is minimally enticed to comply with σ^{NWIGP} if both conditions are satisfied. Since the conditions for sequential rationality are independent of beliefs, any consistent beliefs trivially extend the behavior to sequential equilibrium.

2 Proofs and Intuition

2.1 Proof of Lemma 1

Proof. Call $\delta_i^{min}(g)$ the lowest value of δ that would satisfy a condition for player *i* given network *g*. The comparisons follow straightforwardly from the conditions for full cooperation presented above. Specifically:

1) [From conditions ruling out deviations via (iii) and (viii)] Given network g^A , for $i, j \in A$, $k \in B$: $\frac{\#N_i^{r^T}}{n-1} > \frac{\#N_j^{r^T}}{n-1} \Rightarrow \delta_i^{min}(g^A) < \delta_j^{min}(g^A)$.

2) [From conditions ruling out deviations via (ii), (iv) and (v)] Given network g^A , for $i, j, k, l \in A$, $\min_k \left\{ \frac{\#N_k^{r^T}(g \setminus i)}{n-1} \right\} > \min_l \left\{ \frac{\#N_l^{r^T}(g \setminus j)}{n-1} \right\} \Rightarrow \delta_i^{min}(g^A) < \delta_j^{min}(g^A).$

3) [From conditions ruling out deviations via (iii) and (viii)] For $i \in A$, with network g^A and $i' \in A$ with network $g^{A'}$, $j \in out - group$, $\min_i \left\{ \frac{\#N_i^{rT}}{n-1} \right\} > \min_{i'} \left\{ \frac{\#N_{i'}^{rT}}{n-1} \right\} \Rightarrow \delta^{min}(g^A) < \delta^{min}(g^{A'})$.

4) [From conditions ruling out deviations via (ii), (iv) and (v)] For $i, k \in A$ with network g^A and $i', k' \in A$ with network $g^{A'}, \min_{i,k} \left\{ \frac{\#N_k^{rT}(g^A \setminus i)}{n-1} \right\} > \min_{i',k'} \left\{ \frac{\#N_{k'}^{rT}(g^{A'} \setminus i')}{n-1} \right\} \Rightarrow \delta^{min}(g^A) < \delta^{min}(g^{A'}).$

2.2 Proof of Propositions 1, 2, and 3

The result follows immediately from the conditions for full cooperation above and the example in the text. The conditions make clear that the binding constraint on full cooperation is the reach in the network of the most peripheral group member; the example shows why this

⁵Since the condition must hold for the minimum $\#N_{j-i}^{rT} \ \forall i, j \in A$, it must hold for the minimum $\#N_{i-j}^{rT}$, and the minimum $\#N_{i-j}^{rT}$ is at least as small as the minimum $\#N_i^{rT}$. Hence satisfying the equilibrium condition in terms of $\#N_{i-j}^{rT}$ implies satisfying it in terms of $\#N_i^{rT}$.

position is not identical to network density. The conditions can be rearranged to establish the comparison for networks of arbitrary size in Proposition 2, and the upper bound on the probability of cross-group interactions in Proposition 3.

2.3 Intuition for Corollary 1

The conditions for full cooperation bind for the most peripheral players if there are no controlling players, and bind for either controlling player i whose reach in the network $g \mid i$ is lower than the reach of the most peripheral in network g or the most peripheral in network g when there exist controlling players. Integrated networks have no controlling players (ruled out by the requirement that integrated networks contain two distinct paths between any pair of nodes). Hence, the binding conditions for full cooperation hinge on the most peripheral in the network g. Since integrated networks are those with a maximum value of the minimum rT - neighborhood in the network, no other network with the same number of links could have less peripheral peripheral nodes. Hence, an integrated network is optimal for enforcing cooperation for a given number of links.

3 Controlling Players

Definition (Controlling Players). Let #inf(g) be the number of shortest paths in g that are infinite, i.e. the number of pairs of nodes that have no path in g that connects them. Call player i a Controlling Player of network g if $\#inf(g) < \#inf(g\backslash i)$.

That is, a player is a controlling player if removing that player from the network increases the number of pairs of nodes that cannot reach each other in the network.⁶

Figure 1 contains an example of controlling players. Player 2 is a controlling player in the left network in Figure 1.

A controlling player is well positioned to defect against an in-group member because the message that his victim would spread about him is restricted in its reach through the network. If player 2 defects against player 1, player 1 would tell his neighbors, but his only neighbor is player 2, who will by assumption not participate in spreading the word about his own offense. $\#N_1^{rT}(g\backslash 2) = 0$, leaving just 1 himself to punish 2 if presented with the opportunity. Here 2 controls the access the rest of the group has to 1's information, and can exploit this to defect against 1 with small chance of future punishment.

⁶Players incident to bridging links are one instance of controlling players. A controlling player may have high betweenness centrality, but need not, as Figure 1 demonstrates.

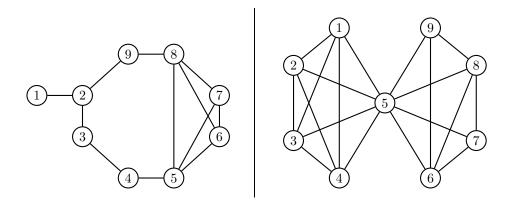


Figure 1: Example of controlling players. On the left, 2 is in a controlling position and has an incentive to defect against 1. On the right, 5 is in a controlling position.

In the right network of Figure 1, player 5 is a controlling player. If 5 defects against 2, 2 tells all of his non-5 neighbors, but the information stops there. 5 sits at a crucial position in the network, controlling the spread of information from one region to another, ensuring that some members of his group are permanently kept out of the loop. If instead a different player, say 1, defects against 2, that information can reach the rest of the network because 1 is not a controlling player.⁷

Controlling individuals pose a problem for full cooperation because news of their in-group defections will never be known about by everyone else in the group.⁸ Controlling individuals also place a limit on the extent to which the improvements discussed in the next section can improve cooperation.

4 Improving Cooperation

4.1 Increasing r

Lemma 1 uncovers important sources of improvement to full cooperation. First, if news spreads more quickly, cooperation is weakly (and often strictly) easier to enforce:

⁷Player 5 connects two regions of the network. Such network positions, including bridges, are thought to be valuable in social networks because they connect diverse social groups and may allow access to novel information that can help in areas like job search (Granovetter, 1973). The present model reminds us that this unique access has a downside: nodes that bridge also wield control over the spread of information, and in some strategic contexts this control can be used to the detriment of the group.

⁸Links adding redundant paths to those blocked by a controlling player would be beneficial, which is related to the closure result of Lippert and Spagnolo (2011) in a different setting.

Corollary (Faster Spreading News is Better). Given g^A, α, β, p and $T, \delta^{min}(r') \leq \delta^{min}(r)$ for r' > r.

The intuition is that when r is larger, news spreads along more links between each round of play, which means weakly more people hear about an offense each period. The larger is r, the weakly larger is expected punishment, and so the weakly smaller the minimum discount factor is that could support full cooperation.

The rate of information transmission affects how well groups can enforce cooperation: a group can enforce cooperation more easily when messages spread more rapidly. Relatedly, groups can support a greater volume of interethnic interactions that are cooperative when messages spread more rapidly. Communities that experience a boon to communications technology should find themselves in a better position to enforce intra- and interethnic cooperation; communities that experience setbacks in their communications technology should find themselves in a worse position to enforce cooperation. This intuitive result is masked by models that presume networks to be complete.

4.2 Increasing T

The relationship between punishment length and cooperation is less straightforward. First, note that if networks were complete so that everyone learned directly and immediately about every other in-group member's interactions, if cooperation were possible, punishment lasting a single round would be at least as effective as punishment that lasts any larger (but finite) number of rounds:

Lemma (Long punishment unnecessary in complete networks). When g^A is complete, if, for some $\alpha, \beta, p, r, \delta, \exists T$ which satisfies the conditions for full cooperation, then T = 1 satisfies the conditions. Moreover, for α, β, p and r, the minimum δ that would support full cooperation in equilibrium for T > 1 is larger than the minimum delta that would support full cooperation in equilibrium for T = 1,

$$\delta^{\min}(T>1) > \delta^{\min}(T=1).$$

Proof. When g^A is complete, $\min_{i,j} \left\{ \frac{\#N_{j-i}^{rT}}{n-1} \right\} = 1 \forall i, j \in A$ and any rT. The conditions for full cooperation then become

$$\frac{\alpha - 1}{(1 - p)(1 + \beta)} \leqslant \delta^T \tag{3}$$

and

$$\frac{\beta}{(1-p)(1+\beta)} \leqslant \delta^T.$$
(4)

Now for a set of parameters α, β, p, δ , since $\delta < 1$, δ^T is decreasing in T, so if $\exists T$ which satisfies the above conditions, T = 1 does. Moreover, since T only enters into the conditions via δ^T , larger T increases the minimum δ required to support full cooperation.

Finite punishment longer than a single round turns out to be not just unnecessary but *harmful* in the case of complete networks because it delays additional punishment that those contemplating defecting a second time in a row would face.

Real world communication networks tend to not be complete; instead, some possible links are absent so that some people receive news second-hand or third-hand or more.⁹ Considering realistic, incomplete networks recovers the potential usefulness of long punishments:

Corollary (Long punishment can be helpful in realistic networks). When g^A is incomplete, for α, β, p, r , it can be that $\exists T > 1$ which satisfies the conditions for full cooperation, and which requires a lower minimum δ than T = 1. In fact, \exists incomplete networks g such that the conditions for full cooperation can only be satisfied for T > 1.

When networks are complete, a longer punishment phase punishment phase (longer T) only serves to delay the last round of punishment. Since this round is the only round relevant to the binding defection– one in which a player defects in a round immediately following his previous defection– then increasing T has the sole consequence of delaying punishment. Since $\delta < 1$, the larger T, the more punishment is attenuated and hence enforcing full cooperation becomes more difficult.

When networks are incomplete (the more realistic case), a longer punishment phase has two consequences: once again it delays the last round in a punishment phase, but it also weakly (and often strictly) increases the number of people who would participate in that round of punishment. The larger T, the more time news of a defection has to spread through an incomplete network, reaching more and more people. The latter can dominate the former and make enforcing cooperation strictly easier as T increases.

Long punishment phases increase the amount of expected punishment by both increasing the number of time periods in which a person is eligible for punishment *and* by increasing

⁹See Larson and Lewis (2016) for an example. Some have argued that humans have an innate upper limit on the number of meaningful relationships they can maintain (Dunbar, 1992); groups with more members than this upper bound naturally have incomplete networks.

the number of people who know that punishment is deserved. This second coordinating role is masked when communication is assumed to be instantaneous.¹⁰

5 The Downside to Ethnic Heterogeneity

As discussed in the text, the fundamental reason out-group interactions are a problem is not that the out-group is not tasked with punishing in-group members. The problem is that information does not reach one group from the other. In fact, if the out-group were tasked with punishing while the communication barrier between the two groups persisted, cooperation would still be easier to enforce if interactions between the two groups were limited. Communication barriers make interactions with other groups reduce a group's ability to enforce cooperation, and consequently create gains from isolation.

To see this, consider a more general setup in which a population has a communication network subdivided into x components C_1, \ldots, C_x with members $\{1, \ldots, n\}$, $\{n + 1, \ldots, 2n\}$ $, \ldots, \{(x-1)n+1, \ldots, xn\}$, respectively.¹¹ This could be an area containing x different ethnic groups. We have so far been considering the case of x = 2 so that there are two distinct sets of people, though we have been adding an additional constraint, that people in one of the sets of people cannot recognize people in the other set.

Now, relax the recognizability constraint so that anyone in any component can recognize anyone in any other component. Suppose that people still encounter each other at random and they make use of an in-group policing strategy where everyone regards everyone else, all xn people, as being part of the in-group. In other words, anyone can punish anyone else if they learn about an offense. Figure 2 shows a toy example with x = 3, xn = 24.

Consider the best case scenario for cooperation: fix r and T to be large enough so that given the other model parameters, the reach of news is as wide as possible before the end of the punishment phase.¹² In this setup, all 24 group members in Figure 2 are charged with punishing anyone else if they know punishment is deserved. Suppose 1 defects against 16. In the best case scenario, news could reach everyone in C_1 and C_2 . No further increase in r or T could increase the reach of news beyond the two components, which means no one in C_3 would know to punish. An even more problematic scenario arises when someone in

¹⁰Real world punishment does vary in duration. Some blood feuds demand single instances of homicide as punishment, some demand more (Boehm, 1984). Some groups of traders punish cheaters with a one-time fee, some charge cheaters with long-term exile (Greif, 1993).

¹¹Components have the same number of nodes to simplify matching probabilities. None of the results here require identically-sized components.

¹²Take $rT = \max_x \{ diam(C_x) \}.$

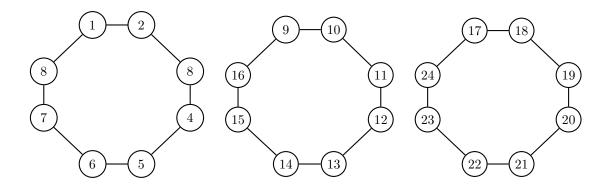


Figure 2: Example group with three components, C_1, C_2 and C_3 .

 C_1 defects against someone else in C_1 . Then at most everyone in C_1 could know, but no one in either C_2 or C_3 could. Communication barriers preventing news in one component from reaching other components limit the maximum number of people who could punish misdeeds.

This setup reveals that the greater the number of individuals who reside outside of a component, the worse the individuals inside that component are able to enforce cooperation in any interactions. This suggests one way that ethnic heterogeneity can undermine full group cooperation. If news relevant to social sanctioning spreads within but not across ethnic groups, then the presence of other ethnic groups with which individuals interact reduces a group's ability to enforce in-group cooperation. The larger the number of individuals outside of one's ethnic group, as when the area is highly heterogeneous, the worse the group is at enforcing both across and within-group cooperation. Heterogeneity dilutes the efficacy of social sanctions.

Proposition (Dangers of Heterogeneity). An ethnic group *i*'s maximum punishment for an in-group offense is decreasing in $\sum_{j \neq i} \#C_j$.

Proof. The maximum probability of punishment occurs when everyone within a component immediately learns of an in-group defection. Consider a component C_i with $\#C_i$ members. The expected probability of in-group punishment depends on the probability of being matched with an in-group member, $\frac{\#C_i}{n-1}$. This is increasing in the number of other individuals within the component, and hence decreasing in the number of other individuals outside the component, $n - \sum_{j \neq i} \#C_j - 1$.

Not only does this logic help to make sense of the empirical difference between cooperation

in ethnically heterogeneous and homogeneous areas, but it also reveals a general incentive to isolate interactions. When barriers prevent news from reaching some, components would be better off only interacting with fellow members of the component. Let p^{own} be the probability that a player is matched with a member of his own component, so that he is randomly matched with a member of any other component with probability $1 - p^{own}$.

Proposition (Gains from Isolation). Groups with components C_1, \ldots, C_x , $x \ge 2$, are best able to enforce cooperation when $p^{own} = 1$.

Proof. Suppose a group with xn members has a network g separated into components $C_1, \ldots, C_x, x \ge 2$, each with n members. Let p^{own} be the probability that a member of a component plays another member of his component. Everyone can recognize everyone, and everyone is tasked with punishing anyone they know has defected. Given α and β , $\delta^{min}(rT)$ takes its minimum when $rT > \max_i \{ diam(C_i) \}$. Fix $rT > \max_i \{ diam(C_i) \}$. When a player defects against a member of his own subgroup, his expected probability of punishment is a mixture of $1 * p^{own}$ and $0 * (1 - p^{own})$. This probability is maximized, and hence δ is minimized, when $p^{own} = 1$.

Even when people can recognize each other and are tasked punishing each other, divisions in the communication network generate incentives to isolate interactions. This means that if information can get stuck in some circles, due to language barriers or cultural preferences to confide in some over others, or norms to communicate with only some others, those circles do best when they never interact with anyone outside those circles. When they do interact outside those circles, enforcing cooperation is more difficult.

6 Extension: Preferential Matches

The results in the article text assume that any two players within group A are equally likely to encounter one another to play the Prisoner's Dilemma. The network determines who discusses outcomes of these encounters with whom– people gossip with network neighbors– but has no bearing on the probability of encountering any particular in-group member. Here I show that, except in an extreme, knife-edge case, interacting more frequently with network neighbors does not change the key result about peripheral members or integrated networks. Network reach rather than network density continues to be the relevant metric for comparing networks. In the article, the probability that player $i \in A$ encounters any other player $j \in A$, conditional on playing someone in A, is $\frac{1}{n-1}$. Suppose instead that, conditional on playing an in-group member, players are more likely to play their network neighbor than anyone else. Call q_i^{nei} the conditional probability that player i plays one if i's network neighbors, $j \in N_i(g)$, and set $q_i^{nei} > \frac{1}{n-1}$. Call q_i^{oth} the conditional probability that i plays a nonneighbor, $j \notin N_i(g)$. Since $q_i^{nei} > \frac{1}{n-1}$, then $q_i^{oth} < \frac{1}{n-1}$.¹³

6.1 Consequences for Expected Punishment

First, consider the consequences of preferential matches for the expected probability of punishment for different types of defection. In a situation of preferential matches (PREF), some types of defections would incur more expected punishment than they would in a situation of uniform random matching (NOPREF). Suppose *i* is considering defecting against the outgroup. Under *NOPREF*, the conditional probability of punishment for an out-group defection would be

$$\frac{1}{n-1} \# N_i^{rT},$$

whereas under PREF, the same probability would be

$$q_i^{nei} \# N_i^{rT}.$$

Since $q_i^{nei} > \frac{1}{n-1}$, expected punishment for out-group offenses is greater for all *i* under *PREF*. In other words:

Result 1. Out-group defections are less likely under preferential matches.

The larger q_i^{nei} , the greater the conditional probability of punishment, and the greater the difference between *PREF* and *NOPREF*.

The difference between NOPREF and PREF is a bit more subtle in the case of in-group defections. Consider *i*'s expected punishment for a defection against one of his network neighbors, $j \in N_i$. Under NOPREF, the conditional probability of punishment for an

¹³Given q_i^{nei} , $q_i^{oth} = \frac{1-q^{nei}}{n-1-\#N_i}$. However, whether these precise values will be feasible for all *i* when matched simultaneously depends on the network structure. Pinning down precise sets of matching probabilities for all players or specifying a Poisson meeting process that could accommodate them adds little to the intuition that this section is intended to convey. For now, simply consider preferential matching from the perspective of a single player *i* with $q_i^{nei} > \frac{1}{n-1}$ and $q_i^{oth} < \frac{1}{n-1}$.

in-group defection against $j \in N_i$ would be

$$\frac{1}{n-1} \# N_j^{rT}$$

and the same under PREF would be

$$q_i^{nei} \# (N_j^{rT} \cap N_i) + q_i^{oth} \# (N_j^{rT} \setminus N_i).$$

The probability under PREF can be larger; this is more likely to be the case when the extent of overlap between j's rT – neighborhood and i's neighborhood is high.

Result 2. In-group defections against network neighbors can be less likely under preferential matches. The result depends on the extent to which a player's and his neighbors' network neighborhoods overlap.

So far preferential matches make out-group defections less likely, and can make in-group defections less likely. However, preferential matches open a third category of possible defections: defections against in-group members far away in the network. In fact, given that i is more likely to play his network neighbors, defections against an in-group player j far from i and i's neighbors in the network become *more* tempting. News of these defections spread from j through the network. If j is far from i's neighbors, the in-group members whom i plays especially frequently are unlikely to hear about his offense. Defecting against those outside of one's neighborhood becomes more profitable.

Specifically, for a player j such that $N_j^{rT} \cap N_i = \emptyset$, the conditional probability of punishment that i expects from defecting against j under NOPREF is

$$\frac{1}{n-1} \# N_j^{rT}$$

and under PREF is

$$q_i^{oth} # N_j^{rT}$$

Since $q_i^{oth} < \frac{1}{n-1}$, the expected probability of punishment is strictly less under *PREF*.

Result 3. In-group defections against individuals far away in the network are more likely under preferential matches.

6.2 Robustness of Density v. Integration Result

Furthermore, probability of punishment under PREF depends on $\#N_j^{rT}$. This means that the most tempting targets outside of one's neighborhood are the most peripheral in-group members. A peripheral in-group member outside of one's network neighborhood is especially unlikely to reach one's neighbors with news of the offense, posing the greatest temptation. In fact, the binding case for full cooperation under preferential matches is the case of temptation to defect against peripheral in-group members far away in the network. This suggests that the integration result still holds:

Result 4. Even under preferential matching, integrated networks are optimal.

In fact, the closer q becomes to 1 without reaching it, the greater the temptation to defect against peripheral ingroup members outside of one's network neighborhood. This temptation is only remedied in the limiting case in which q = 1, i.e. players exactly never encounter anyone except network neighbors. In this extreme case, players never have the chance to defect against someone far away because they never encounter someone far away: players only ever have the opportunity to defect in ways that would be known by the in-group members they encounter. In this limiting case, peripheral network positions are unproblematic because players always play someone to whom they are connected; however, in this case, the density of the ethnic group is not only not a sufficient condition for cooperation, but it is irrelevant to cooperation.

Adding links to the network would have no bearing on incentives to defect against the out-group or to defect on in-group members far away (opportunities to do the latter would never be realized). The arrangement of links only affects incentives to defect against network neighbors. The optimal network would be a fragmented one in which every person had only a single neighbor. The fragmented pairs could perfectly punish wrong-doing against the out-group and against each other. Equally good would be networks with perfectly overlapping networks– networks fragmented into fully-connected cliques. However, the interaction network of disjoint pairs containing only $\frac{n}{2}$ would be just as cooperative as the network of larger cliques with more links.

While this limiting case may be informative for ways that networks splinter or change over time, and perhaps is informative about gradual subdivisions of larger populations into ethnicities and clans, it is probably a poor representation of real ethnic groups living together in a village. Daily life may require interactions with many others in the village. These interactions can in principle occur independent of a network of interpersonal communicationbonds forged due to trust and a willingness to confide (Larson, 2016). If interactions are not in practice independent of these bonds as the article text assumes, they are likely to be only partly constrained by them– a relatively higher probability of encountering someone trusted enough to share information with– rather than fully determined by them – the impossibility of interacting with anyone else.

The above intuition can be summarized as the following:

- Even when people are more likely to encounter their neighbors in a network than other ingroup members, peripheral network positions still pose problems for full cooperation. So long as people occasionally encounter non network neighbors, then the more likely they are to encounter their network neighbors, the more tempting it is to defect against the peripheral ingroup players far away in the network. Integrated networks are still optimal.
- 2. In the limiting case in which people *only* encounter their neighbors in a network and exactly never encounter any other ingroup members, peripheral network positions are unproblematic. However, in this case, it is not the density of the ethnic network, but local pockets of density that improve cooperation.

7 Elaboration of the Relationship between Network Density and Cooperation

Here I further elaborate on the relationship between network density and full intra- and interethnic cooperation. The density of a network is the ratio of links present in the network to the total possible number of links, which for an undirected network of n nodes is $\frac{n(n-1)}{2}$. For two networks with the same number of nodes, comparing the number of links present is equivalent to comparing density.

Lemma 1 suggests the following:

Result 5. Intra- and interethnic cooperation are easiest to enforce when the network g is perfectly dense.

That is, when every possible link is present, peer sanctions are maximally effective at enforcing cooperation: full cooperation can be enforced in equilibrium for the lowest possible value of the discount factor. In this case, every coethnic would learn about any offense immediately, making the set of punishers every other coethnic. In this sense, density is helpful for cooperation.

However, realistic communication networks among ethnic groups are not perfectly dense.¹⁴ Instead, individuals confide in only a subset of their group. The important question for assessing prospects for cooperation empirically is whether groups with imperfectly-dense networks (i.e. incomplete networks) can be compared in terms of their respective density.

Although perfect density is optimal in a sense, it is not the case that for incomplete networks the denser network is better for enforcing cooperation:

Result 6. Given incomplete networks g^A and g^B , g^A being denser than g^B does not imply that cooperation is easier to enforce among A, and cooperation being easier to enforce among A does not imply that g^A is denser than g^B . Greater density is neither necessary nor sufficient for cooperation to be easier to enforce.

This is the same result that is reported in the main article. Because peripheral and controlling network positions bind, a network can be denser and yet enforcing full cooperation can be more difficult, and for a network such that enforcing full cooperation can be easier, that network can be less dense. Density is simply not the relevant metric for comparing groups with incomplete (and hence realistic) networks.

Now, it is the case that if links were arranged optimally, a larger number of links reduces the minimum discount factor that supports the enforcement of full cooperation in equilibrium. In other words, for a given number of nodes, a larger number of links makes enforcing cooperation easier given that the links are arranged optimally. As established in the article text, the optimal arrangement is one that maximizes the minimum rT – neighborhood in the network and eliminates controlling positions.

Result 7. For a set of nodes of size n, and number of links e and e' such that e' > e, enforcing cooperation on a network with the optimal (perfectly integrated) arrangement of e' links is weakly easier than enforcing cooperation on a network with the optimal arrangement of e links.

If a network could be constructed from scratch and the links could be forged optimally, then doing so with more links to arrange will result in a network on which enforcing full inter- and intra-ethnic cooperation is easier. However, it is important to separate this result from the one before it. Simply because we could design a more cooperative network if

¹⁴See Larson and Lewis (2016).

we had more links to work with does not mean that if we observe two groups, the group with the denser network will be more cooperative. Whether the denser network will be more cooperative depends on how peripheral the most peripheral positions are and how controlling the controlling positions are.

Finally, we can consider the addition of links to existing networks.

Result 8. Adding links to existing networks with a fixed set of nodes weakly improves cooperation.

Formally, given α, β, p, r and T, If $g^{A'} \supset g^A$, then $\delta^{min}(g^{A'}) \leq \delta^{min}(g^A)$.

Proof. Given α, β, p, r and T, If $g^{A'} \supset g^A$, then $\exists i \in A$ such that $N_i^{rT}(g^{A'}) > N_i^{rT}(g^A)$, and for all other $j \neq i$, $N_j^r(g^{A'}) \ge N_j^r(g^A)$. Then the result follows.

This suggests that for a given ethnic group, forging new meaningful lines of communication can improve cooperation by increasing the reach of news that allows people to punish wrongdoing.¹⁵ Manipulating the communication network may be challenging but is not impossible; examples are discussed in the article text. This result also suggests that removing links from a given group weakly hurts its prospects for cooperation. When relationships deteriorate or when something prevents two people from making use of their regular channel of communication, enforcing cooperation becomes more difficult.

As the proof makes clear, the additional link among a fixed set of players matters because it increases at least one person's rT-neighborhood without reducing anyone else's. However, the conditions for full cooperation also suggest that the weak inequality will often be equality: the only time an additional link changes the ease of enforcing full cooperation is when it affects the rT-neighborhood of the player with the smallest rT-neighborhood. Increasing others' rT-neighborhoods has no bearing on the ease of enforcing full cooperation since if they could be made to be cooperative without the additional link, they could be made to be cooperative with the additional link. Full inter- and intra-ethnic cooperation depends on the most peripheral network positions.

However, the addition of links to existing networks that are accompanied by the addition of nodes can strictly *decrease* cooperation. To see this, consider a set of nodes N and a new entrant i. Suppose the new entrant will be added with a link to an existing member, so the network grows by one link and one individual. The individual to whom the new entrant links

¹⁵Of course not every strategic context will have this property. Here, additional links forge new lines of communication that weakly improve the chances of social sanction. In other strategic settings, like those that exhibit free riding or collective action problems, this result would not necessarily hold.

will necessarily become a controlling player; in the left network of Figure 1 above, node 1 could be a new entrant and 2 is the existing member to whom he is linked. 2 then becomes a controlling player. 2 would have a new, strictly greater incentive to defect against 1 since if he did so, he could block 1's report of this misbehavior from reaching any other coethnics. In summary:

Result 9. Adding links to existing networks while also adding nodes can strictly reduce cooperation.

Hence, the relationship between density and cooperation is subtle. Perfectly dense networks are the most cooperative. When networks are imperfectly dense, density is not the correct metric by which to compare the ease of cooperation for the two networks. Networks can be denser and strictly worse at enforcing cooperation, and networks can be better at enforcing cooperation while being less dense. What matters is the arrangement of links: if links were arranged in such a way as to maximize the reach of the most peripheral players and eliminate controlling positions, then having more links would be better. But this is different from saying that we can observe two networks and compare them in terms of their relative densities. Adding links without adding new nodes always weakly improves cooperation (though the relation is only strict if the links improve the reach of peripheral players or reduce the control of controlling players); adding links while also adding nodes can strictly reduce cooperation.

8 Extension: Alternate Information Environment

In contrast to the information environment in the body of the paper in which news of defections only spreads from victims, consider one in which all players honestly announce whether they have been the victim *or* the perpetrator of a deviation.¹⁶ When news is truthful by assumption, there is no difference between in-group interactions that are unobservable and those observable to neighbors. Information spreads from every possible source in such an environment; even still, some networks face greater barriers to cooperation, and the barriers still stem from peripheral network positions.

In this environment, messages take the following form: when *i* deviates from σ^{NWIGP} in a round with in-group member *j* or an out group member, *i* sends a message $m_{i,i,t} = \{i, t\}$ to

¹⁶This environment also captures the scenario in which actions are verifiable so that cooperative interactions generate some evidence that a player could produce to show his neighbors that he behaved cooperatively in his last interaction and failure to produce this evidence reveals that punishment is deserved. Thanks to an anonymous reviewer for suggesting this interpretation.

himself and his neighbors and j sends a message $m_{j,i,t} = \{i, t\}$ to himself and his neighbors which spreads through the network at rate r. Call $M_{i,t}$ the set of individuals about whom i has received messages by the start of time t about rounds that have occurred since t - T.

This information environment differs only in that messages are also transmitted from the perpetrators of coethnic offenses in addition to the victims of such offenses. The network comparisons that result are similar. Lemma 1 in this environment becomes:

Given α, β, p, r and T we can make the following comparisons:

1) In group A with network g^A , person *i* can be more easily enticed to be cooperative in out-group pairings than person *j* when

$$\frac{\#N_i^{rT}}{n-1} > \frac{\#N_j^{rT}}{n-1}.$$

2) In group A with network g^A , person i can be more easily enticed to be cooperative in in-group pairings than person j when

$$\min_{k} \left\{ \frac{\#(N_i^{rT} \cup N_k^{rT})}{n-1} \right\} > \min_{l} \left\{ \frac{\#(N_j^{rT} \cup N_l^{rT})}{n-1} \right\}$$

for $k, l \in A$.

3) For group A, network g^A satisfies the conditions for cooperation with an out-group B more easily than network $g^{A'}$ when

$$\min_{i} \left\{ \frac{\# N_{i}^{rT}}{n-1} \right\} > \min_{i'} \left\{ \frac{\# N_{i'}^{rT}}{n-1} \right\}$$

for $i \in A$ with network g^A , $i' \in A$ with network $g^{A'}$.

4) For group A, network g^A satisfies the conditions for cooperation among in-group members more easily than a network $g^{A'}$ when

$$\min_{i,k} \left\{ \frac{\#(N_i^{rT} \cup N_k^{rT})}{n-1} \right\} > \min_{i',k'} \left\{ \frac{\#(N_{i'}^{rT} \cup N_{k'}^{rT})}{n-1} \right\}$$

for $i, k \in A$ with network $g^A, i', k' \in A$ with network $g^{A'}$.

The only difference is that controlling players are no longer a problem. When both victims and perpetrators of defections spread messages (perhaps because misbehavior is observable to network neighbors), bottleneck positions in the network no longer confine messages.

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